Problem Bank for Sections of 10/1 and 10/6

October 6th, 2020

Group 1

Find the derivative of each function defined as follows.

1. $$y = 5x^4 + 9x^3 + 12x^2 - 7x.$$

2. $$y = -100\sqrt{x} - 11x^{2/3}.$$

3. $$f(t) = \frac{t^3 - 4t}{\sqrt{t}}$$

4. $$h(x) = (x^2 - 1)^3$$

Group 2

Application to Life Scientists: Optimal Foraging Using data collected by zoologist Reto Zach, the work done by a crow (a bird) to break open a whelk (large marine snail) can be estimated by the function

$$W = \left(1 + \frac{20}{H - 0.93}\right) H, \tag{1}$$

where H is the height (in meters) of the whelk when it is dropped. *Source: The Mathematics Teacher.*

1. Find dW/dH

2. One can show that the amount of work is minimized when $dW/dH = 0$. Find the value of H that minimizes W.

3. Interestingly, Zach observed the crows dropping whelks from an average height of 5.23 m. What does this imply?

Group 3

Application to Economics: Profit

An analyst has found the a company's costs and revenues in dollars for one product are given by

$$C(x) = 2x \quad \text{and} \quad R(x) = 6x - \frac{x^2}{1000}, \tag{2}$$

respectively, where x is the number of items produced.

1. Find the marginal cost function.
2. Find the marginal revenue function.
3. Using the fact that profit is the difference between revenue and costs, find the marginal profit function.
4. What value of x makes marginal profit equal 0?
5. Find the profit when the marginal profit is 0.

Group 4

Find the equation of the tangent line to the graph for

1. $y = -3x^5 - 8x^3 + 4x^2$ at $x = 1$, and
2. $y = -x^{-3} + x^{-2}$ at $x = 2$.
3. $y = x^2 - 5$ at $x = 2$. For this find the x intercept of this tangent line. This is a reasonable approximation of $\sqrt{5}$, why is this?

Group 5

Find the derivative of each function as follows

1. $$f(t) = 12(2t^4 + 5)^{3/2}$$
2. $$y = (x^3 + 2)(x^2 - 1)^4 \tag{3}$$

Extra Problems

1. **Application to Economics: Marginal Average Cost**
   Suppose that the demand function is given by $\bar{C}(x) = C(x)/x$, where x is the number of items produced. Show that the marginal average cost function is given by
$$\bar{C}'(x) = \frac{xC'(x) - C(x)}{x^2}. \tag{4}$$

2. **Application to Economics: Revenue**
   Suppose that at the beginning of the year, a Vermont maple syrup distributor found that the demand for maple syrup, sold at \$15 a quart, was 500 quarts each month. At that time, the price was going up at a rate of \$0.50 a month, but despite this, the demand was going up at a rate of 30 quarts a month due to increased advertising. How fast was the revenue increasing?